Fractal Methods for Modeling and Forecasting of Currency Crises

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Results of fractal analysis of daily exchange rate fluctuations of floating currencies for a 10-year period are presented. It is shown that monetary crashes are usually preceded by prolonged periods of abnormal fractal exponent. Regression relations between duration and magnitude of currency crises and the degree of distortion of monofractal patterns of exchange rate dynamics are found and have been used as a basic ingredient of a forecasting technique which provided correct in-sample predictions of monetary crisis magnitude and duration over various time scales.

Keywords**:** fractal analysis, risk, currency crisis, forecasting, prediction

Short-term evolution of exchange rates of many national currencies is defined by the results of inter-bank trades coordinated by the Foreign Exchange (FOREX) system. This regulation mechanism is usually considered by international financial institutions as the most appropriate for the conditions of open markets influenced by the globalization process [1]. However, the model of floating exchange rates faced a sequence of large monetary crashes during the period from 1993 to 2003, which has revealed disadvantages of currently used methodological basis of crisis prevention as well as the lack of theoretical understanding of stability principles underlying a long-term behavior of complex financial and economic systems [2 - 4].

The possibility of using fractal analysis methods for estimating long-term stability of complex dynamical systems is based on the theory of self-organized criticality, a relatively new framework developed in statistical physics for modeling catastrophic events in nonlinear systems with many coupled degrees of freedom [5]. In the present work, this approach is applied to investigate fractal temporal behavior of a large number of floating exchange rates, including robust currency dynamics in economically developed countries and unstable exchange rate fluctuations in developing countries with emergent market economies many of which underwent large-scale monetary crashes. The results obtained help better understand mechanisms of dynamical instabilities of national monetary systems leading to abrupt devaluation, and provide new quantitative criteria of pre-crisis currency dynamics summarized in a phenomenological model that can be used for forecasting a characteristic magnitude of exchange rate fluctuations during the active crisis phase.

This study is based on nonstationary fractal analysis of time series of daily values of independent floating exchange rates in 31 countries [6]. The exchange rates were expressed in normalized units equal to the exchange rate of the U.S. dollar in local currency units. The data were downloaded from the historic database provided by OANDA corporation and covered a time interval from May, 1993 to April, 2003.

It is known that currency exchange rates as long as some other macroeconomic parameters usually exhibit a multiscale temporal structure characteristic of random fractal processes such as fractional Brownian motion [7-9]. Absolute values of increments of these processes obey scaling relations in the form

$$\left\langle \left| x(t+\Delta t) - x(t) \right| \right\rangle = \frac{1}{r^H} \left\langle \left| x(t+r\Delta t) - x(t) \right| \right\rangle, \quad (1)$$

where $x(t)$ is the time series under study, Δt is a time shift, and H is the Hurst exponent [10]. Formula (1) expected to be valid for all values of the scaling factor $r > 0$ within the limits of fractal dynamics of $x(t)$. Samples of $x(t)$ on the $x-t$ plain are characterized by a non-integer coastline dimension $D = 2 - H$ taking on values between 1 and 2. The amount of autocorrelations in the time series of $x(t)$ increments is estimated by the quantity

$$C = 2^{2H-1} - 1. \quad (2)$$

According to this relation, the correlations are absent if H = 0.5 (D = 1.5). This implies that the increments are statistically independent at any time scale. If $x(t)$ is a fluctuating macroeconomic parameter, statistical independence of its increments corresponds to the conditions of free and efficient information exchange between market agents driving the market to the point of its optimal self-organization associated with steady dynamical balance between supply and demand [9].

Due to long-term financial and economic processes affecting the currency dynamics, fractal structure of exchange rates may undergo slow reorganization leading to a time-dependent fractal dimension. To quantify such dynamics, we developed a technique of nonstationary fractal dimension estimation based on the algorithm of detrended fluctuation analysis (DFA) [11]. In a nutshell, the technique is a sliding-window generalization of the DFA method. For each time instant t, scaling properties of studied time series between the instants $t-T$ and t are described by the time-dependent DFA function

$$F_t(\tau)=\sqrt{\frac{1}{(\tau+1)(T-\tau+1)}\sum_{k=t-T}^{t-\tau}\left(\sum_{i=k}^{k+\tau}\left(y_i-\left[a(i,\,i+\tau)+b(i,\,i+\tau)\times i\right]\right)^2\right)} \tag{3}$$

in which τ is the time scale, $a(t_1,\,t_2)$ и $b(t_1,\,t_2)$ are the regression coefficients obtained from the linear approximation of the accumulated time series

$$y_i=\sum_{k=t-T}^{i}\left(x_k-\frac{1}{T+1}\sum_{m=t-T}^{t}x_m\right),\quad t-T\le i\le t, \tag{4}$$

within the interval $[t_1,t_2]$. Time variables used in (3) and (4) are integer. For fractal time series obeying the relation (1), the DFA function (3) to scale as

$$F_t\sim\tau^{\alpha(t)}. \tag{5}$$

The dependence (5) has been considered as a definition of the time-dependent DFA exponent α connected with current estimation of fractal dimension through α =3–*D*.

Two informative scaling ranges of exchange rate fluctuations (τ = 4–30 and 30–90 days) have been studied based on relations (3)-(5). These ranges were found to represent short-term and long-term tendencies in currency dynamics [6], which should be analyzed prior to monetary crashes in countries with unstable monetary systems. These ranges were characterized by DFA exponents α_1 and α_2 considered as functions of time. Mean values of these exponents were calculated by averaging over time series of $\alpha_1(t)$ and $\alpha_2(t)$ constructed with window T=360 over time intervals of relatively uniform currency dynamics. For unstable currencies, such intervals were identified as time periods before and after active phase of crises; for stable currencies, mean α_1 and α_2 values were calculated during the entire period of analysis. Comparison of exponent values between different country groups was performed using the Student's test with statistical significance $p>0.95$.

To investigate a possibility of predicting size and duration of monetary crashes, the following two indicators of the distortion of fractal scaling of exchange rate fluctuations prior to crisis have been considered:

$$S^{+}_{\alpha>1.75}=\sum_{t=t_0-L}^{t_0}\left(\alpha_2(t)-1.75\right)\times\theta\left(\alpha_2(t)-1.75\right),\qquad S^{-}_{\alpha<1.25}=\sum_{t=t_0-L}^{t_0}\left(1.25-\alpha_2(t)\right)\times\theta\left(1.25-\alpha_2(t)\right) \tag{6}$$

Here L is crisis duration, θ is the Heaviside step function ($\theta(a)$=1 for $a\ge 0$, $\theta(a)$=0 for $a<0$), t_0 – time of crisis beginning. Cumulative indicators $S^{+}_{\alpha>1.75}$ and $S^{+}_{\alpha>1.75}$ quantify an accumulated deviation of α_2 parameter outside the interval [1.25, 1.75]. As shown below, this interval is associated with normal DFA exponent variations in countries with stable monetary systems, correspondingly. We have studied statistical correlation of these indicators with the duration L of the crisis as well as with its normalized magnitude W given by the ratio of standard deviations of exchange rate fluctuations before the crisis ($t\in(t_0-L,t_0)$) and during its active phase.

Analysis of mean α_1 and α_2 values in different countries has shown that large and systematical deviations of these parameters from the value 1.5 corresponding to the efficient market conditions [6, 9] correlate with a decrease of long-term dynamical stability of floating exchange rates. This dependence allowed us to develop a classification for selecting groups of countries with high and low efficiency of national currency systems. The selected groups coincide with groups of countries characterized by high and low macroeconomic stability (Table 1).

The results shown in Table 1 suggest that stable exchange rates dynamics in economically developed countries (group N) is described by robust and uniform temporal fractal structure. The average values of both short-term and long-term DFA exponents of these currencies are fairly close the value 1.5 predicted by the efficient market hypothesis (EMH). At the same time, nonstationary estimates of the DFA exponents in this group are characterized by reproducibly multiscale structure symmetrical with respect to the 1.5 level.

Table 1. Stable and unstable groups of exchange rate dynamics

	Type of dynamics	**DFA exponents**	**Macroeconomic characteristics**
N	Stable	Normal: α_1=1.500±0.009, α_2 =1.479±0.031	Economically developed countries: *Great Britain, Greece, EU, Canada, New Zealand, Norway, USA, Swiss, Japan, Australia*
D	Marginally stable	Slightly increased: α_1=1.505±0.038, α_2 =1.550±0.055	Developing countries with relatively stable monetary systems: *Israel, Columbia, Chili, South Africa*
H **L**	Unstable, prior to crises	Increased (H) α_1 and α_2: α_1=1.729±0.048*, α_2 =1.952±0.041* Decreased (L) α_2: α_1=1.426± 0.226, α_2 =1.109±0.333*	Developing countries during periods preceding large-scale monetary crashes: *Bulgaria, Brazil, India, Kazakhstan, Mexico, Russia, Rumania, Turkey, Ecuador*
A	Unstable, prior to crises	Decreased α_2: α_1= 1.387±0.181, α_2 =1.261±0.064*	Developing Asian countries before the 1997 monetary crisis: *Indonesia, Malaysia, Singapore, Thailand, Taiwan, Philippines, South Korea*
M	Marginally stable, after crises	Normalization of both exponents: α_1=1.513±0.038, α_2 =1.567±0.032*	Countries from groups H, L and A after crises

* statistically significant difference from group **N**

Four other currencies whose long-term fractal exponent α_2 was slightly but systematically higher that the EMH value refer to developing countries with relatively stable economies (group D). Dynamics of these currencies does not exhibit large-scale crises; however, it often shows periods of large fluctuations and trends not characteristic of stable currencies from group N. Another distinctive feature of group D is significant nonstationarity of temporal evolution of the DFA exponents.

In contrast to groups N and D, time series of groups H, L and A have revealed various types of significant non-fractal distortions during the periods before monetary crashes, with the exponent α_2 being more sensitive to unstable regimes in currency dynamics compared to α_1. It has been found that currency crises are typically preceded by periods of anomalously low or anomalously high α_2 values indicating a considerable reduction of long-term dynamical stability of domestic financial systems. During the post-crisis period, the DFA exponents tend return to their normal values around 1.5, which correlates with the restoration of monetary equilibrium.

The observed effects can be interpreted as a manifestation of direct relation between disturbed fractal scaling of exchange rate fluctuations and a violation of financial conditions for stable currency dynamics leading to large-scale macroeconomic crises.

By comparing time series of exchange rates in economically developed and developing countries, characteristic signatures of nonstationary variations of fractal structure of currency fluctuations under stable and unstable macroeconomic conditions have been revealed. It turned out that values of α_2 exponent of floating currencies which did not undergo crashes during the period of study remained within the limits 1.25–1.75. Its width is approximately equal to doubled standard deviation of DFA exponent variations averaged over group N. The subsequent analysis of unstable exchange rate dynamics has confirmed that this range can be considered as normal. It has been shown that significant distortions of scale-invariant fractal structure of currency dynamics accompanied by prolonged deviations of α_1 and/or α_2 parameters beyond the specified limits indicate a critically unstable regimes of national currency systems. In the absence of adequate protective measures aimed at the restoration of stable pattern of currency fluctuations, such states usually result in large-scale monetary crashes.

Despite a considerable amplitude of DFA exponents fluctuations in group N, they very rarely crossed the limits of the normal range of values during the entire period of study. Group D currencies demonstrated systematic but insignificant violations of this range, which is in agreement with their intermediate position in our classification. In contrast, time series of unstable currencies from groups L, A and H exhibited much more dramatic and frequent deviations of DFA exponents beyond the limits of the normal range of values. Exponent α_2 appeared to be especially sensitive to the approaching currency crisis. Thus, for example, the exchange rates from group H were characterized by persistently increased α_2 values which were significantly higher than 1.75, while the currencies from groups L and A had systematically lowered α_2 values below 1.25. Typically, after the active phase of crisis, the values of α_2 exponent in these currency groups became closer to the value 1.5, and its dynamics became similar to α_2 dynamics in marginally stable currencies of group D.

The revealed dependence presents an opportunity of diagnosing pre-crisis conditions in macroeconomic systems by deviations of α_2 beyond the specified above normal limits. Indeed, we have found that systematic violation of the condition $1.25 < \alpha_2 < 1.75$ can be considered as an indicator of an approaching large-scale crisis whose magnitude depends on the time during which the exchange rate dynamics was characterized by abnormal α_2 values associated with lowered stability of national monetary system. The regression analysis of relationships between characteristics of the crisis (W and L) considered as functions of cumulative fractal parameters $S^-_{\alpha<1.25}$ and $S^+_{\alpha>1.75}$ (6) showing the degree of deviation of exchange rate dynamics from the normal α range has revealed strong statistical correlation between these two groups of variables (Fig. 1-2).

In addition, it has been found that same regression relations $W(S^-_{\alpha<1.25})$ and $W(S^+_{\alpha>1.75})$ with fixed regression coefficients can be used for describing both short-term periods of reversed instability in exchange rate dynamics of economically developed countries and large-scale currency crashes in countries with developing economies. This implies a universal mechanism producing exchange rate instabilities that can be used for predicting parameters of monetary crashes based on the analysis of small events whose number can be sufficient for obtaining quantitative estimates of financial risks and making quantitative statistical estimations.

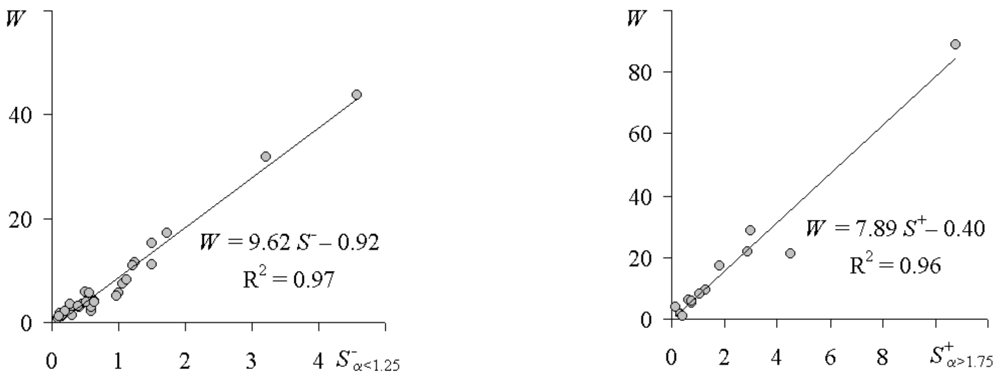

Fig. 1. Normalized crisis magnitude as a function of cumulative fractal indices characterizing the degree of fractal distortions in time series of exchange rate fluctuations with $\alpha_2 < 1.25$ (left) or $\alpha_2 > 1.75$ (right)

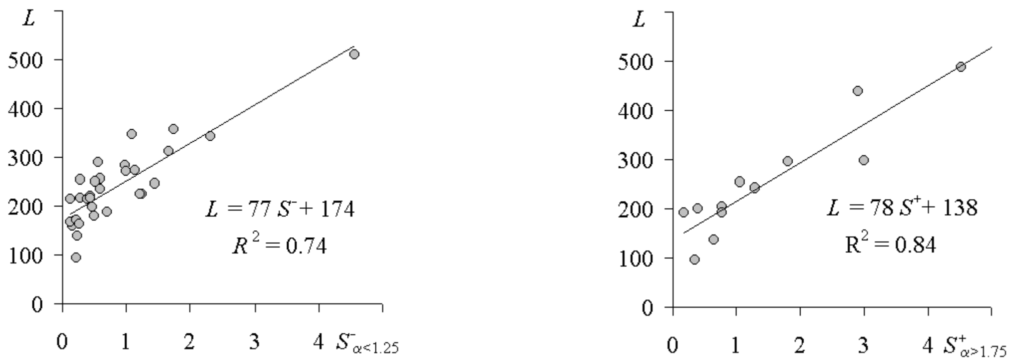

Fig. 2. Crisis duration (in days) as a function of the same parameters as in Fig.1.

Using the results of analysis of correlations between monetary crashes and fractal dynamics of exchange rate fluctuations, we have constructed a regression model allowing quantitative prediction of important crisis parameters (duration L, normalized magnitude W, maximum amount of currency devaluation x_{max}, exchange rate volatility σ_1) using statistical characteristics of currency dynamics before the crisis (cumulative indices $S^-_{\alpha<1.25}$ and $S^+_{\alpha>1.75}$, mean value $\langle x \rangle_0$ and standard deviation σ_0 of currency fluctuations). Basic model equations as well as the values of empirically obtained regression coefficients are shown in Table 2.

Table 2. Fractal regression model of currency crises

Type of fractal distortion before crisis	**Increased DFA exponent** $S^-_{\alpha<1.25}=0,\quad S^+_{\alpha>1.75}\neq 0$	**Decreased DFA exponent** $S^-_{\alpha<1.25}\neq 0,\ S^+_{\alpha>1.75}=0$
Regression equations	$L = K^+_L\, S^+_{\alpha>1.75} + a^+_L$ $W = K^+_W\, S^+_{\alpha>1.75} + a^+_W$	$L = K^-_L\, S^-_{\alpha<1.25} + a^-_L$ $W = K^-_W\, S^-_{\alpha<1.25} + a^-_W$
Expected volatility	$\sigma_1 = \sigma_0\left(K^+_W\, S^+_{\alpha>1.75} + a^+_W\right)$	$\sigma_1 = \sigma_0\left(K^-_W\, S^-_{\alpha<1.25} + a^-_W\right)$
Expected devaluation	$x_{max} = \langle x \rangle_0 + 3\sigma_0\left(K^+_W\, S^+_{\alpha>1.75} + a^+_W\right)$	$x_{max} = \langle x \rangle_0 + 3\sigma_0\left(K^-_W\, S^-_{\alpha<1.25} + a^-_W\right)$
Regression coefficients (p **<0.01)**	$K^+_W = 7.9 \pm 1.3$ $a^+_W = -0.41 \pm 0.86$ $K^+_L = 78 \pm 13$ $a^+_L = 138 \pm 20$	$K^-_W = 9.6 \pm 0.7$ $a^-_W = -0.92 \pm 0.64$ $K^-_L = 77 \pm 7$ $a^-_L = 174 \pm 17$

In case of significant and prolonged α_2 deviation from the normal range leading to non-zero values of $S^-_{\alpha<1.25}$ or $S^+_{\alpha>1.75}$, the constructed model can be used for predicting monetary system response to short-term exchange rate instabilities as well as the range of full-scale crises in the case when such events become inevitable. Such information presents a significant practical interest and, in particular, can help estimate the adequacy of preventive measures applied by national and international financial institutions. If such actions are not accompanied by a restoration of normal fractal structure of exchange rate fluctuations associated with the EMH condition $\alpha_1 = \alpha_2 = 1.5$ or lead to its further breaking-down manifested in growing cumulative indices $S^-_{\alpha<1.25}$ or $S^+_{\alpha>1.75}$, they should be considered as wrong or incomplete. It should be noted that the crisis onset time is left beyond the scope of the model as its statistical prediction requires taking into account specific structural features of monetary systems complemented by a complete sets of external macroeconomic variables.

To estimate the performance of the constructed model, statistics of predicted x_{max} and L values expressed as a percentage of the actual values observed during most significant currency crises which occurred during the studied period has been investigated. On average, the error of x_{max} in-sample prediction was as low as 5 %. Prediction of L was characterized by somewhat larger discrepancies (10%) which may reflect the fact that the crisis length is affected by fiscal measures undertaken by national governments during the period following its beginning stage.

The results obtained indicate a possibility of fractal forecasting of pre-crisis unstable evolution of open macroeconomic systems. They also suggest that the most stable state of such systems is attained when currency fluctuations are characterized by fractal temporal structure consistent with the EMH prediction. Deviations of the DFA exponent from the interval $\alpha = 1.5 \pm 0.25$ provide a quantitative basis for statistically justified classification of long-term exchange rate dynamics which is in a good agreement with macroeconomic situations in studied countries.

It has also been found that currency crises are usually preceded by fairly long periods of "hidden" violation of dynamical stability conditions. During these periods, the DFA exponents are systematically increased or decreased beyond the normal interval of values. After the crisis, fractal exchange rate dynamics self-organizes into a more efficient state with the DFA exponents tending to the EMH value 1.5. The revealed signatures of unstable exchange rate dynamics can be used for forecasting the size and the characteristic time scale of currency crises in countries with essentially different monetary systems including transient currency fluctuations in economically developed countries. In the latter case, the normalized magnitude W should be considered as a predictor for future volatility of currency fluctuations.

The quantitative criteria of unstable currency dynamics derived in this study can be used as a mathematical basis for a new generation of forecasting techniques aimed at the evaluation of dynamical stability of national monetary systems, estimating of currency and financial risks, as well as predicting parameters of large-scale monetary crashes using available historic data on small-scale currency instabilities. This information may be useful for real-time monitoring of dynamical stability of floating exchange rate systems and creating advanced early-warning-system models for currency crisis prevention.

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